

**QP Code : 14535**

(3 Hours)

[ Total Marks : 80

- N.B :** (1) Question no.1 is compulsory.  
(2) Answer any **three** from remaining.  
(3) **Figures** to the **right** indicate marks.

1. (a) Find laplace transform of  $t^3 \cos t$ . 5  
 (b) Find the image of  $|z - ai| = a$  under the transformation  $w = \frac{1}{z}$ . 5  
 (c) Construct an analytic function whose real part is  $e^{2x} (x \cos 2y - y \sin 2y)$ . 5  
 (d) Show that the set of functions  $\cos nx$   $n = 1, 2, 3 \dots$  is orthogonal on  $(0, 2\pi)$ . 5

2. (a) By using Convolution Theorem. Find invese laplace transform of  $\frac{1}{s^2(s+1)^2}$ . 6  
 (b) Find bilinear transformation that maps the points  $2, i, -2$  onto the point  $1, i, -1$ . 6  
 (c) Find Fourier Series for  $f(x) = \cos mx$  in  $(\pi, -\pi)$  where  $m$  is not an integer. Deduce 8

that  $\cos m\pi = \frac{2m}{\pi} \left( \frac{1}{2m^2} + \frac{1}{m^2-1^2} + \frac{1}{m^2-2^2} \dots \frac{1}{m^2-n^2} \right)$  hence show that

$$\sum_{n=1}^{\infty} \frac{1}{9n^2-1} = \frac{1}{2} - \frac{\pi\sqrt{3}}{18}$$

3. (a) Find Complex form of fourier series  $f(x) = e^{3x}$  in  $0 < x < 3$ . 6  
 (b) Using Crank Nicholason method solve  $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$  subject to  $0 \leq x \leq 1$   $u(0, t) = 0$ , 6  
 $u(1, t) = 0$ ,  $u(x, 0) = 100x(1-x)$  taking  $h = 0.25$  in one step.  
 (c) Using laplace transform solve  $(D^2+2D+5)y = e^{-t} \sin t$  when  $y(0) = 0$  and  $y'(0) = 1$ . 8

4. (a) Evaluate  $\int f(z) dz$  along the Parabola  $y = 2x^2$  from  $z = 0$  to  $z = 3 + 18i$  where 6  
 $f(z) = x^2 - 2iy$   
 (b) Find half range cosinc series for 6

$$f(x) = x \quad 0 < x < \frac{\pi}{2}$$

$$= \pi - x \quad \frac{\pi}{2} < x < \pi$$

- (c) Obtain two distinct Laurent's series of  $f(z) = \frac{1}{(1+z^2)(z+2)}$  for  $1 < |z| < 2$  and  $|z| > 2$ . 8

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5. (a) By using Bender Schmidt method solve  $\frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial t}$   $f(0, t) = f(5, t) = 0$ . 6  
 $f(x, 0) = x^2(25 - x^2)$  find  $f$  in range taking  $h = 1$  and upto 5 seconds.

(b) Evaluate  $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$ . 6

(c) Evaluate  $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4 \cos \theta} d\theta$ . 8

6. (a) A string is stretched and fastened to two points distance  $\ell$  apart. motion is started by displacing the string in the form  $y = a \sin\left(\frac{\pi x}{\ell}\right)$  from which it is released at time  $t = 0$ . Show that the displacement of a point at a distance  $x$  from one end at a distance  $x$  from one end at time  $t$  is given by  $y(x, t) = a \sin\left(\frac{\pi x}{\ell}\right) \cos\left(\pi \frac{ct}{\ell}\right)$ . 6

- (b) If  $f(z) = u + iv$  is analytic and  $u - v = e^x (\cos y - \sin y)$  find  $f(z)$  in terms of  $z$ . 6  
 (c) Evaluate: 8

$$L^{-1}(2 \tanh^{-1} s)$$

$$L^{-1}\left[\frac{s}{(s-2)^6}\right]$$